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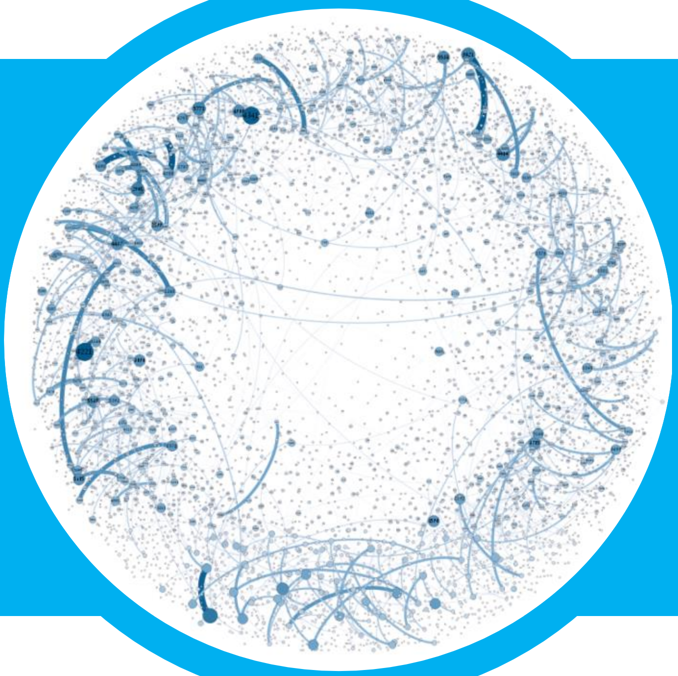
Network GPS – Navigating network dynamics

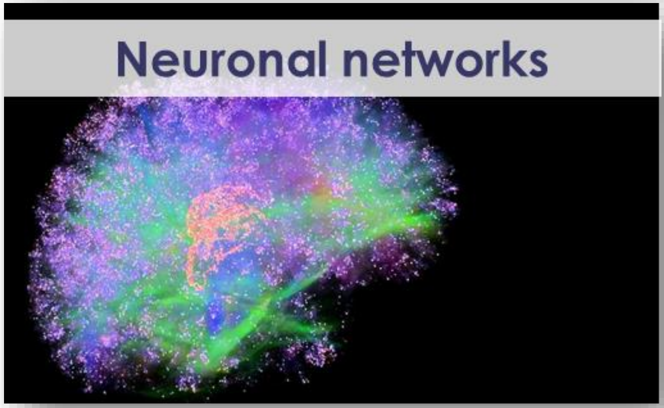
Baruch Barzel
Bar-Ilan University

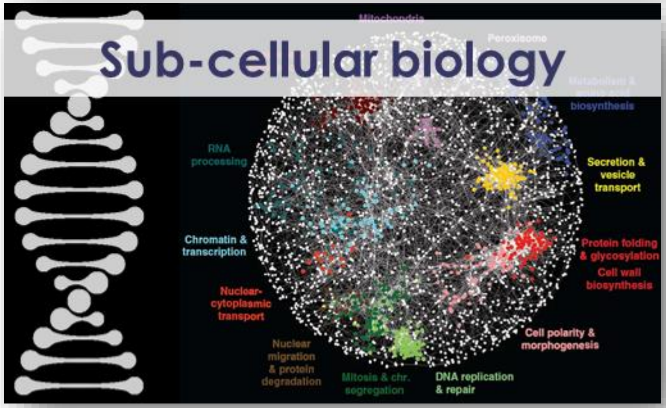


2003 NORTHEAST BLACKOUT

5.5×10^7 People affected
10^2 Fatalities
6×10^9 USD in damages

Crucial role of networks



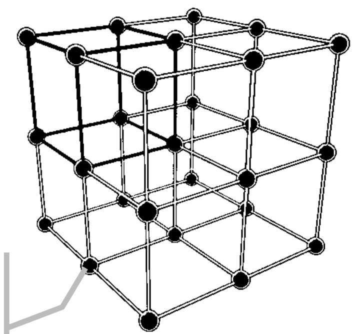


Going beyond mapping

Going beyond mapping

The challenge of networks

Statistical physics comfort zone:



Each node has $k = 6$ nearest neighbors

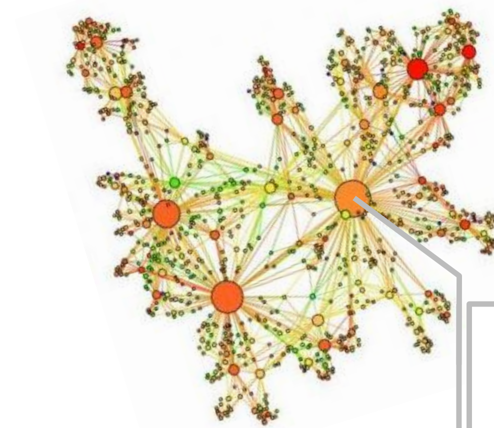
- Low dimensional
- Symmetric structures (lattice or lattice-like)

Finite dimension: $\langle d \rangle \sim \sqrt[\mathrm{Dim}]{N}$

Degree distribution: bounded $P(k) \sim e^{-k}$

Hamiltonian dynamics

Where real networks are:



k spans orders of magnitude

- Disordered and weighted (for now: positive)
- Heterogeneous – Hubs and small nodes

Small world (infinite dimension): $\langle d \rangle \sim \log N$

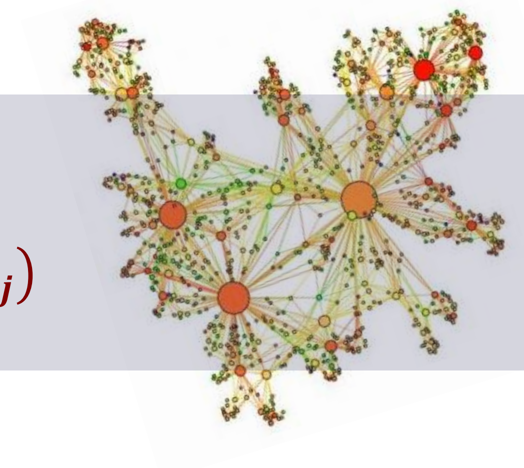
Degree distribution: scale free $P(k) \sim k^{-\gamma}$

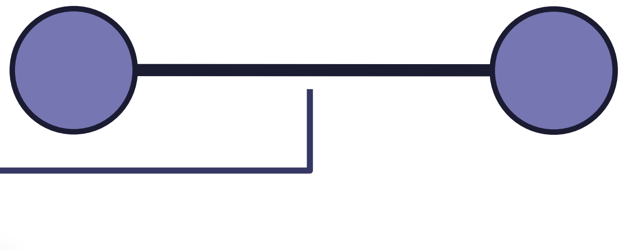
Nonlinear dynamics (sometimes hidden)

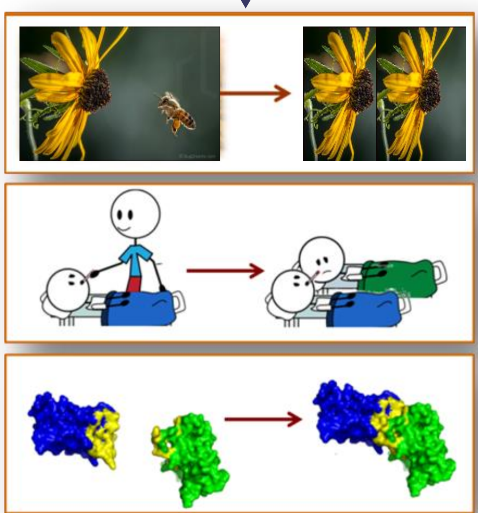
Dynamics layer

$\boldsymbol{x_i(t)}$ → Activity

$$\frac{dx_i}{dt} = M_0(x_i) + \sum_{j=1}^{N} A_{ij} M_1(x_i)\, M_2(x_j)$$







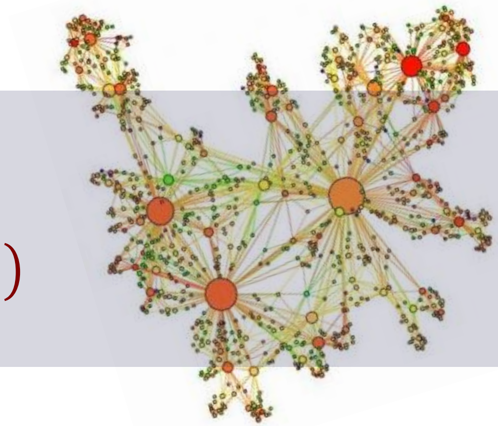
A_{ij} Weighted, directed topology

$x_i(t)$ Concentration, Infection probability, Species abundance

M_0, M_1, M_2 Intrinsic nonlinear interaction mechanisms

Gao, Barzel, Barabási. *Nature* **530**, 307 (2016) • Barzel, Barabási. *Nature Physics* **9**, 673 (2013)

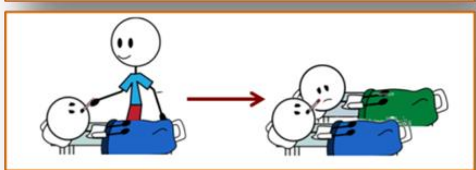
Dynamics layer



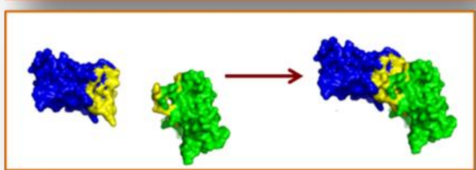
$x_i(t) \rightarrow$ Activity

$$\frac{dx_i}{dt} = M_0(x_i) + \sum_{j=1}^{N} A_{ij} M_1(x_i)\, M_2(x_j)$$

$$\frac{dx_i}{dt} = Bx_i\ (1 - x_i) + \sum_{j=1}^{N} A_{ij} \frac{x_i x_j^a}{1 + x_j^a}$$



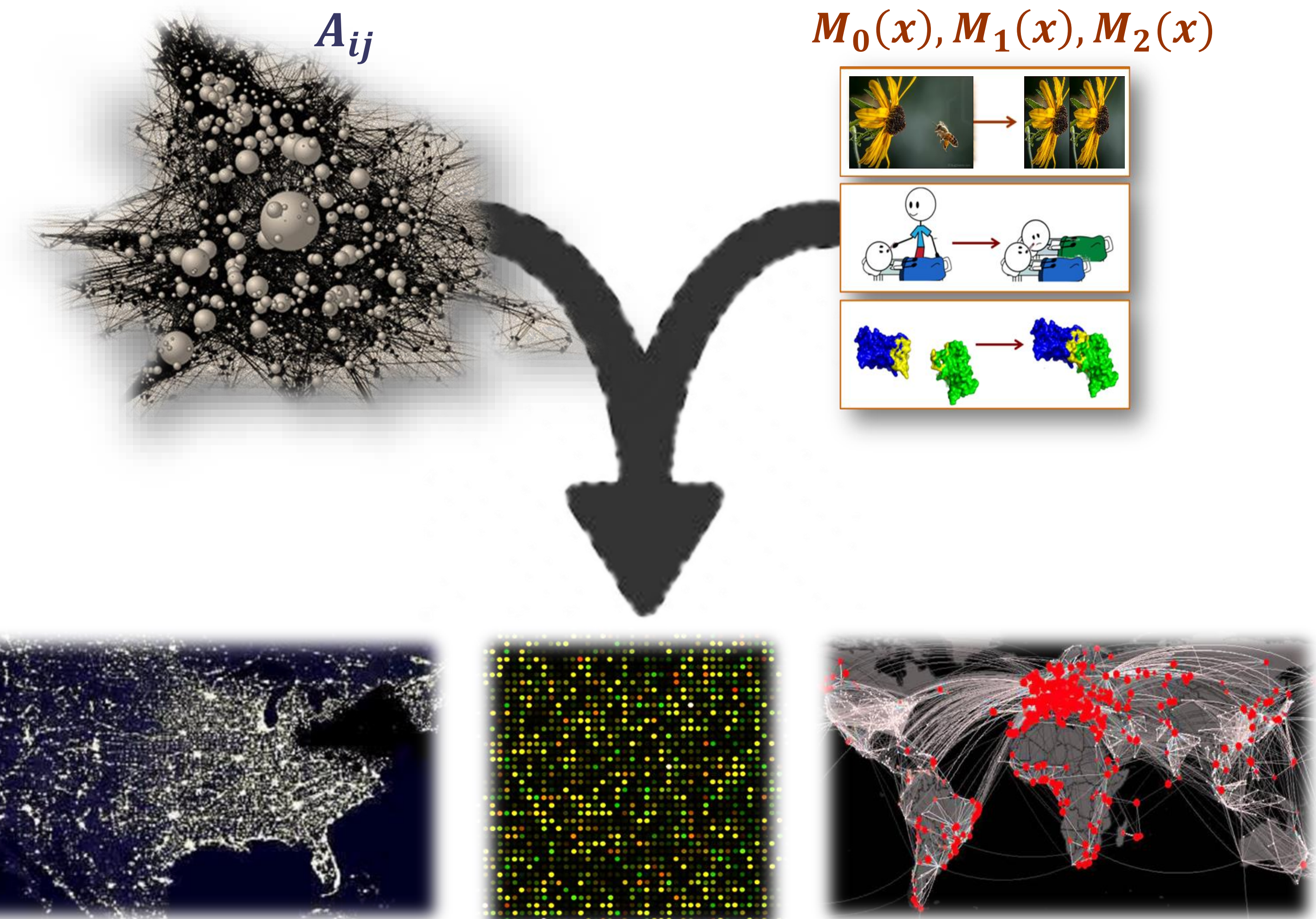
$$\frac{dx_i}{dt} = -Bx_i + \sum_{j=1}^{N} A_{ij}(1 - x_i)x_j$$



$$\frac{dx_i}{dt} = -Bx_i + \sum_{j=1}^{N} A_{ij} \frac{x_j^h}{1 + x_j^h}$$

Gao, Barzel, Barabási. *Nature* **530**, 307 (2016) • Barzel, Barabási. *Nature Physics* **9**, 673 (2013)

Bringing networks to life

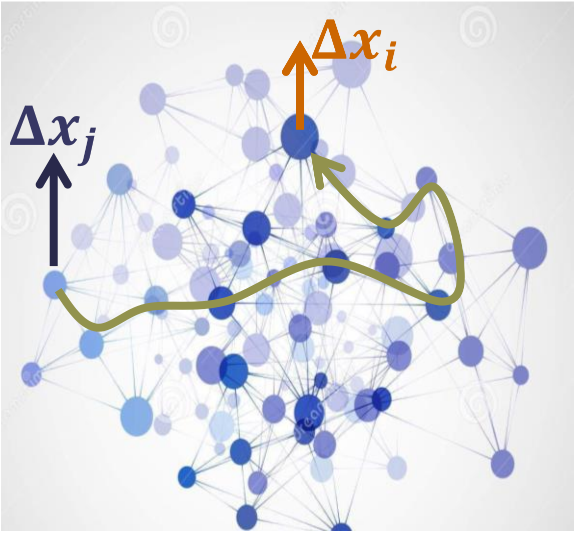


patterns of information spread

Information flow in complex networks

$$\boldsymbol{x_j} \to \boldsymbol{x_j} + \boldsymbol{\Delta x_j} \longrightarrow \boldsymbol{x_i} + \boldsymbol{\Delta x_i(t)}$$

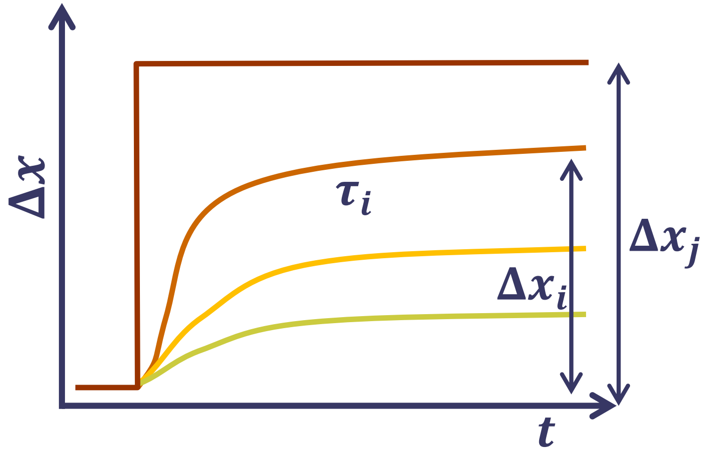
signal response



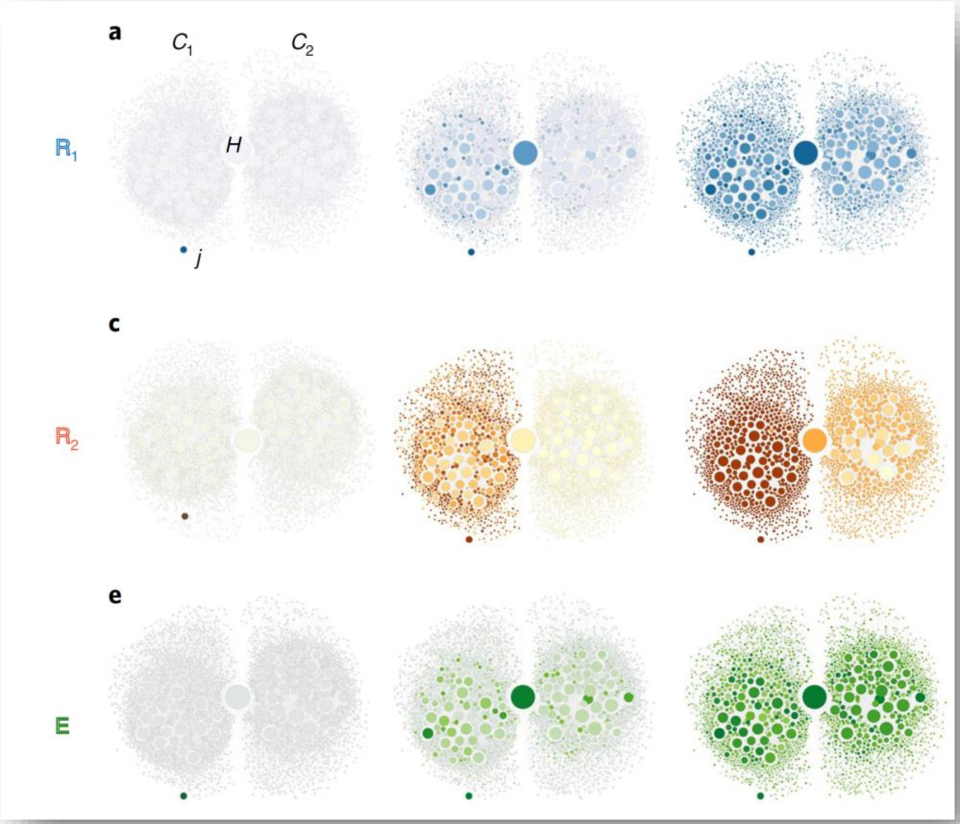
Response patterns to fixed-point perturbations

Information flow in complex networks

- When $T(j \to i)$
- Where $l_{ij}(t)$
- How strongly Δx_i
- How $(\mathcal{F}_i, \mathcal{F}_{ij})$



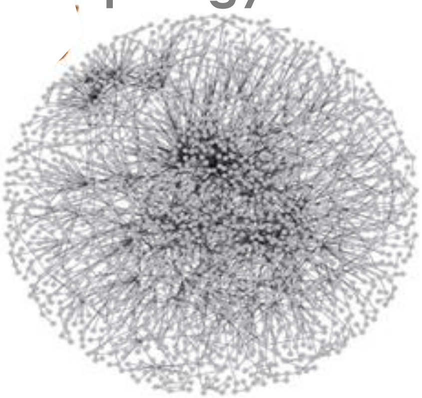
Spatiotemporal propagation



Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

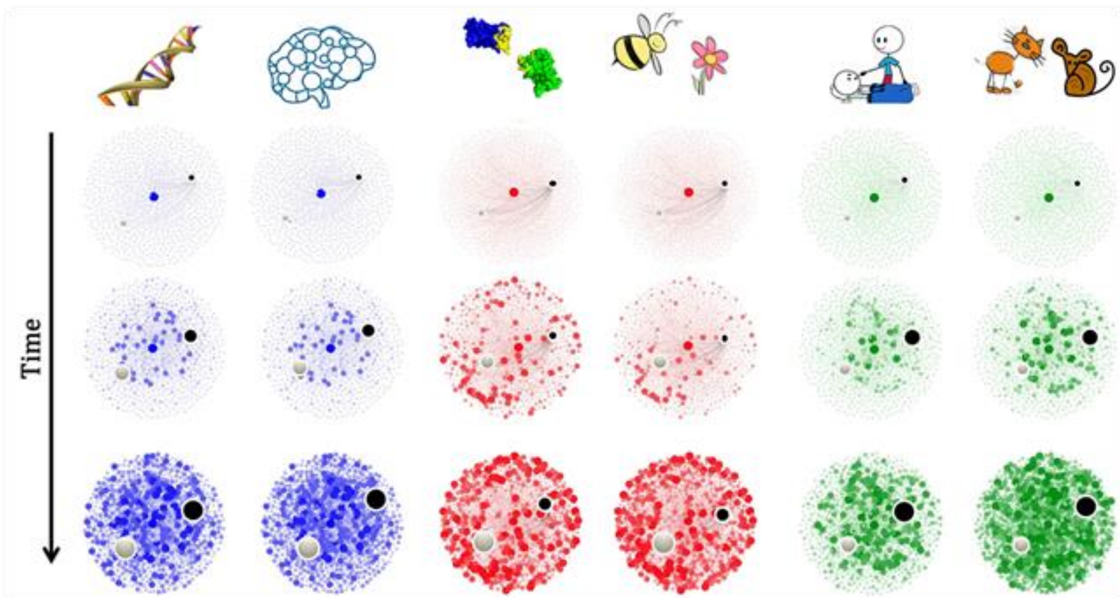
The zoo of propagation patterns

Topology



Identical networks yield visibly distinct propagation patterns

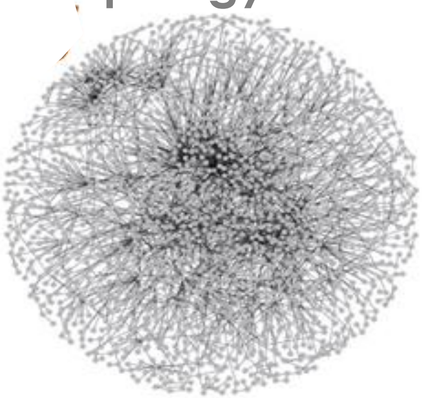
Interaction mechanisms



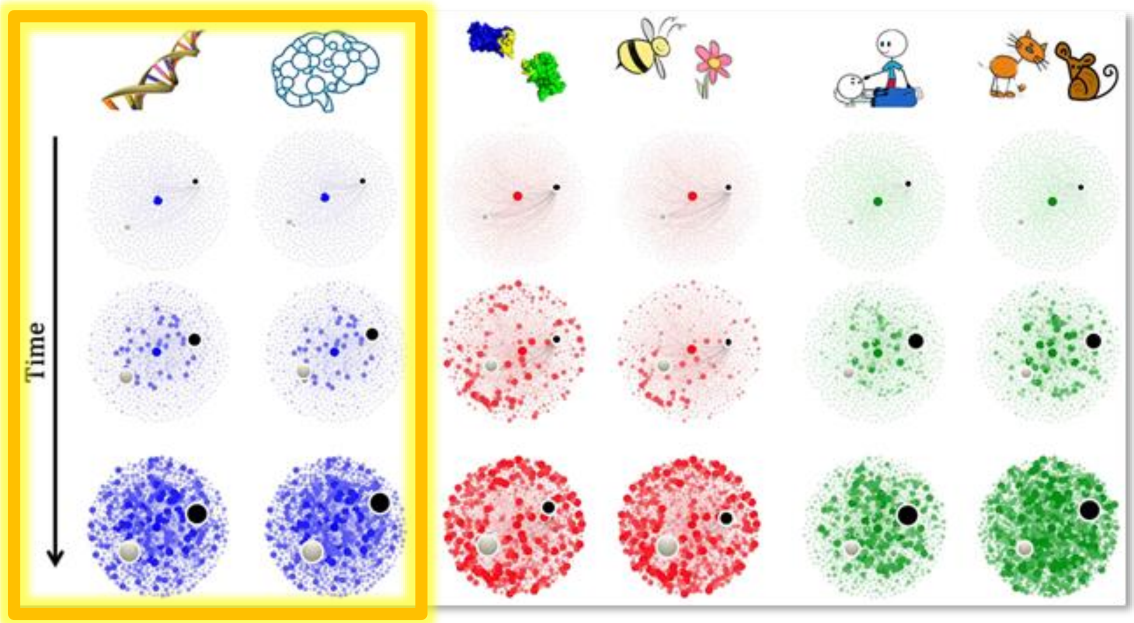
$t \sim 10^{1}$ $t \sim 10^{4}$ $t \sim 10^{-2}$

The zoo of propagation patterns

Topology



Interaction mechanisms



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The zoo of propagation patterns

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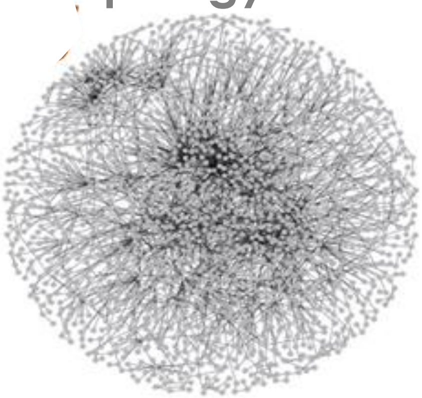

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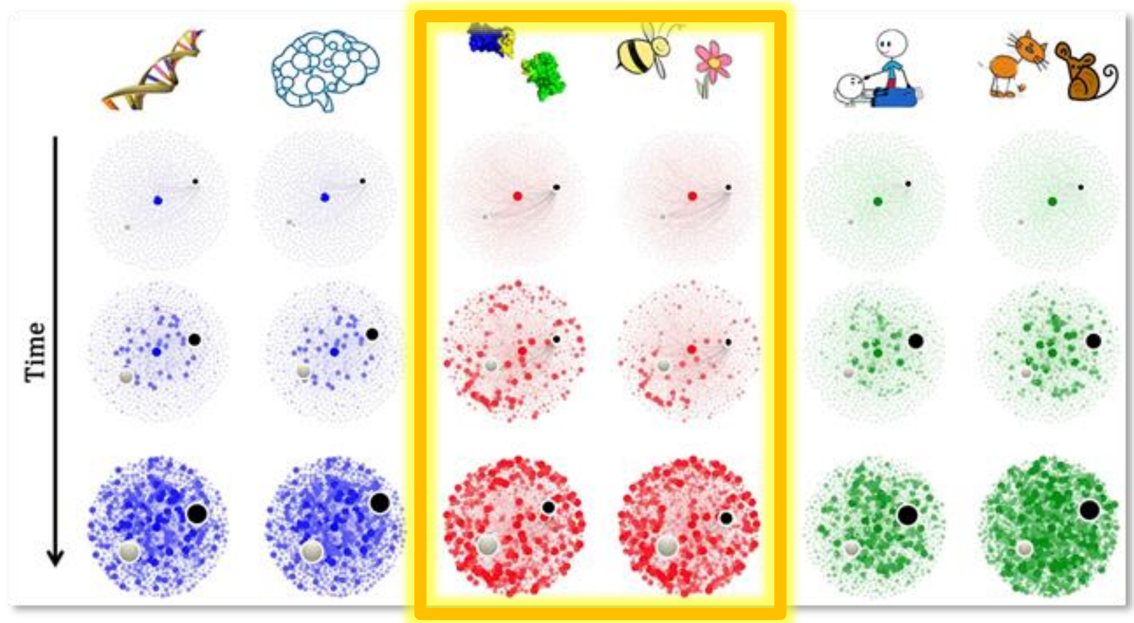

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The zoo of propagation patterns

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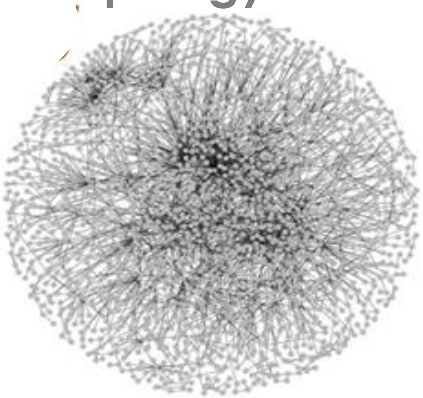
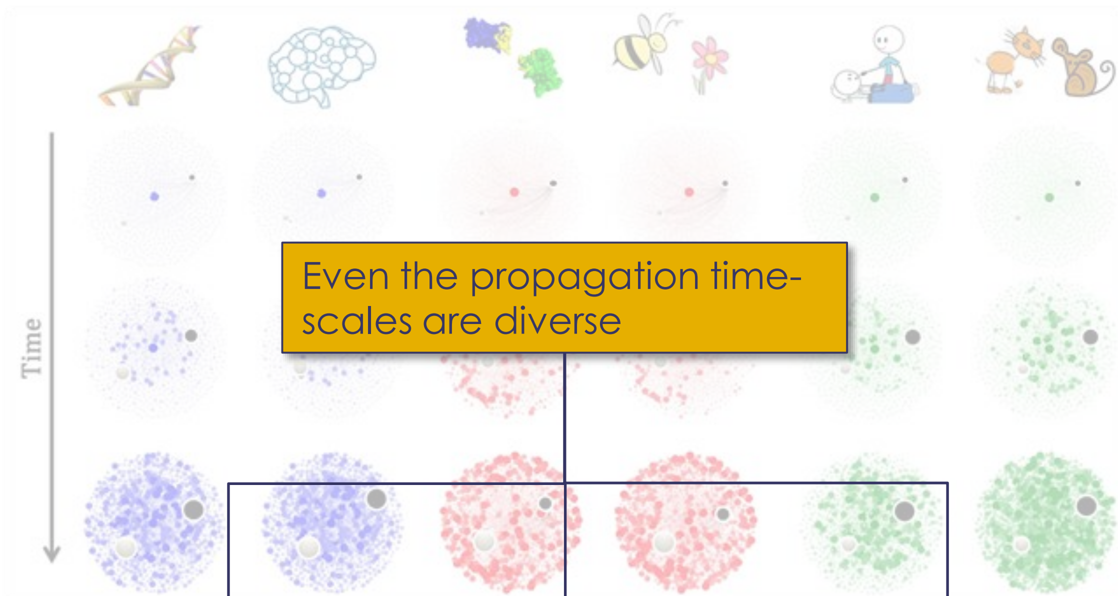


Identical networks yield visibly distinct propagation patterns

Interaction mechanisms



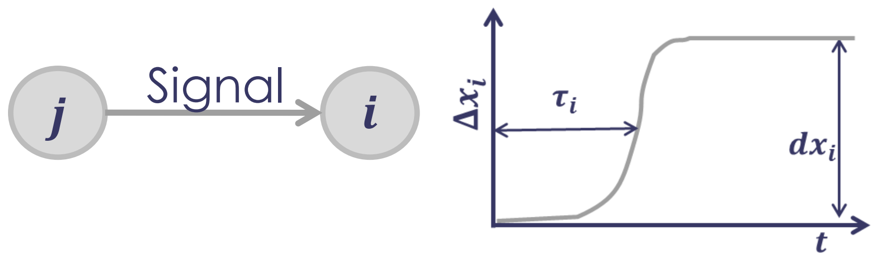
Even the propagation time-scales are diverse

$t \sim 10^1$ $t \sim 10^4$ $t \sim 10^{-2}$

Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

Taming the zoo of propagation patterns

Individual node response time



k_i Node i's weighted degree
τ_i Node i's individual response time

$$\boldsymbol{\tau_i \sim k_i^{\theta}}$$

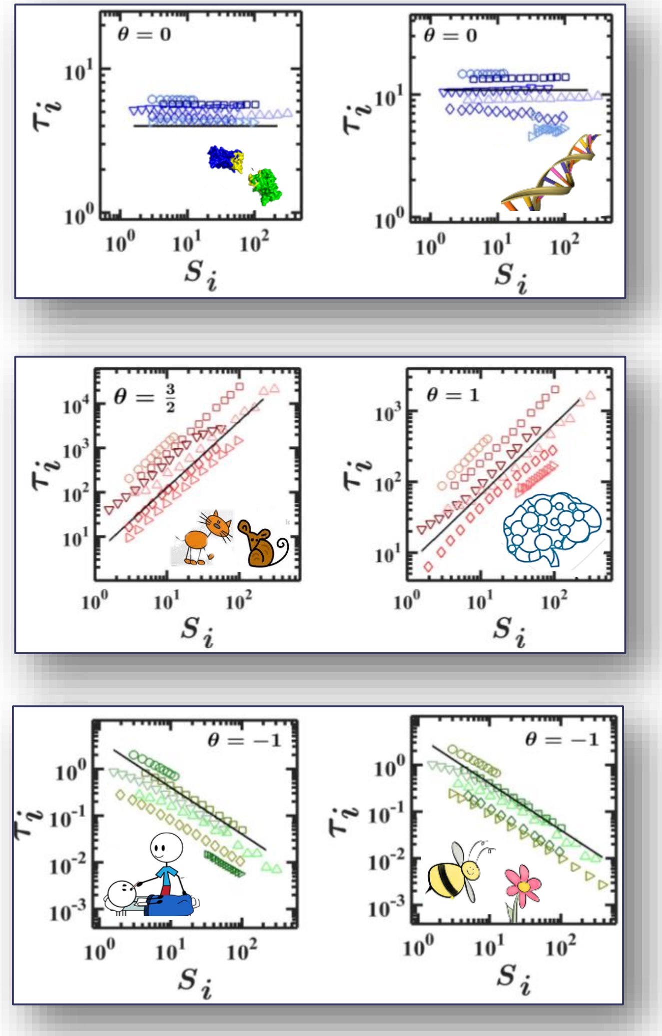
A node's intrinsic response time scales with its weighted degree

Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

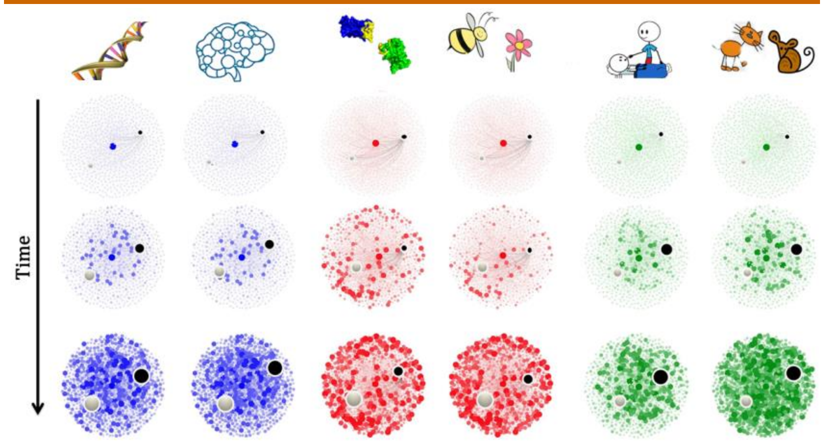
Why should you care?

Universal

$$\tau_i \sim k_i^{\theta}$$



Diverse & unpredictable



Why should you care?

How do we obtain the exponent θ?

$$\frac{dx_i}{dt} = M_0(x_i) + \sum_{j=1}^{N} A_{ij} M_1(x_i)\, M_2(x_j)$$

$$R(x) = -\frac{M_1(x)}{M_0(x)}$$

$$Y(x) = \left(\frac{d\big(M_1(x)R(x)\big)}{dx}\right)^{-1}$$

$$Y\big(R^{-1}(x)\big) = \sum_{n=0}^{\infty} C_n x^{\Gamma(n)}$$

$$\theta = -2 - \Gamma(0)$$

$$\tau_i \sim k_i^{\theta}$$

Why should you care:

θ depends on the system's Dynamics (M_0, M_1, M_2)

Only on leading powers (through $\Gamma(0)$)

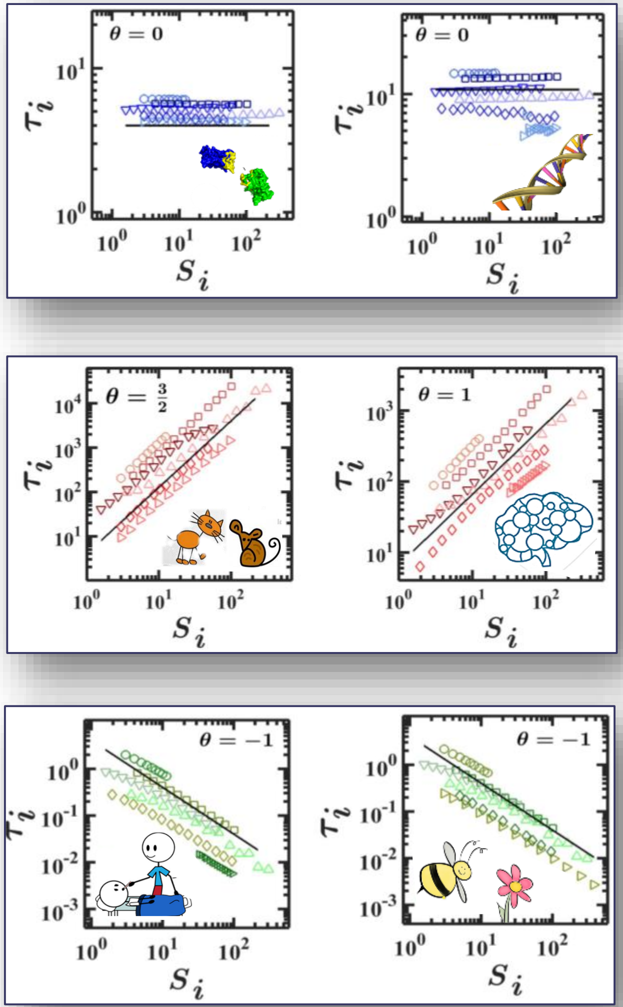
Not on coefficients (C_n)

Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

Universality

Universal

$$\tau_i \sim k_i^{\theta}$$



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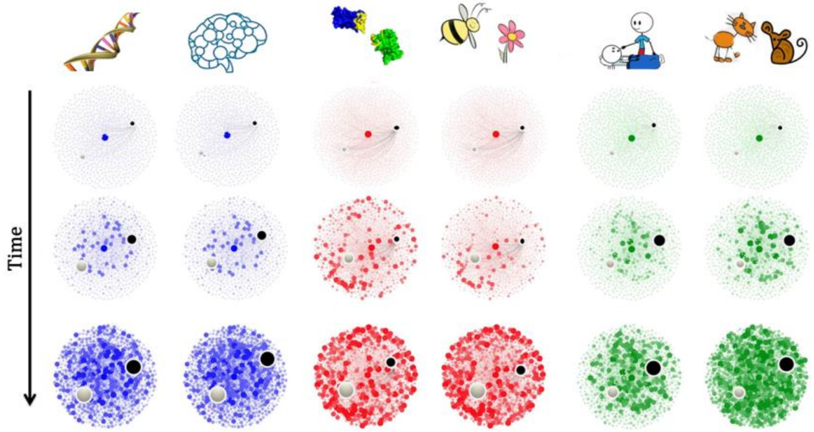


Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

Dynamic insight

Response time
Dynamic observable of interest.

$$\tau_i \sim k_i^{\theta}$$

Dynamic determinant
Mapping topology into dynamics

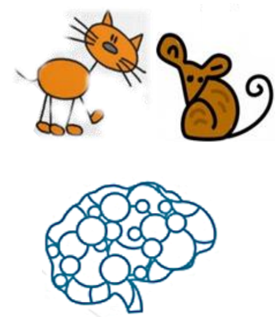
Weighted degree
Known topological element

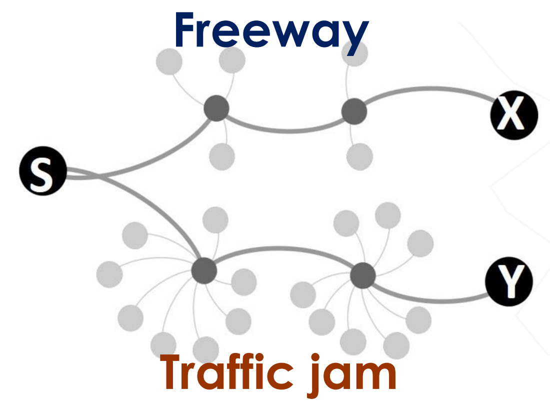
Network GPS – how signals navigate the network

$$\tau_i \sim k_i^{\theta}$$

$$\theta > 0$$

Hubs
=
Bottlenecks





$$\theta < 0$$

Hubs
=
Free flow

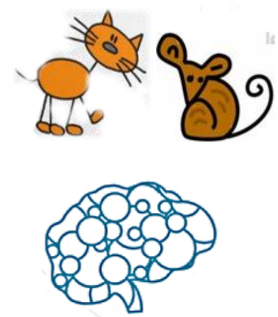
Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

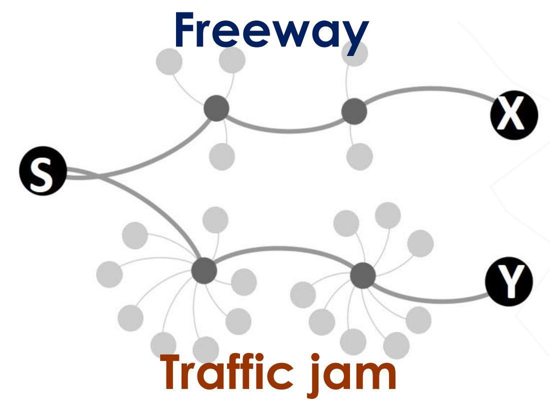
Network GPS – how signals navigate the network

$$\tau_i \sim k_i^{\theta}$$

$$\theta > 0$$

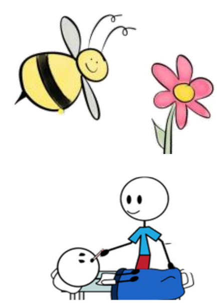
Hubs
=
Bottlenecks

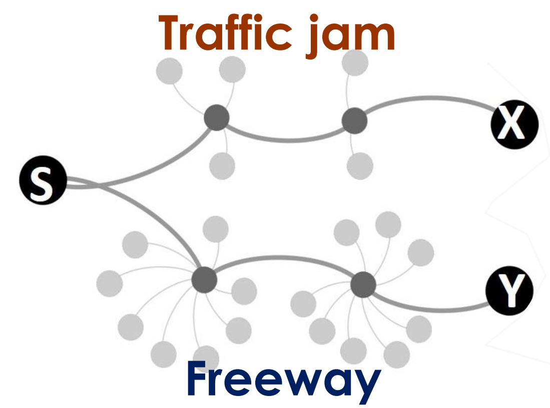




$$\theta < 0$$

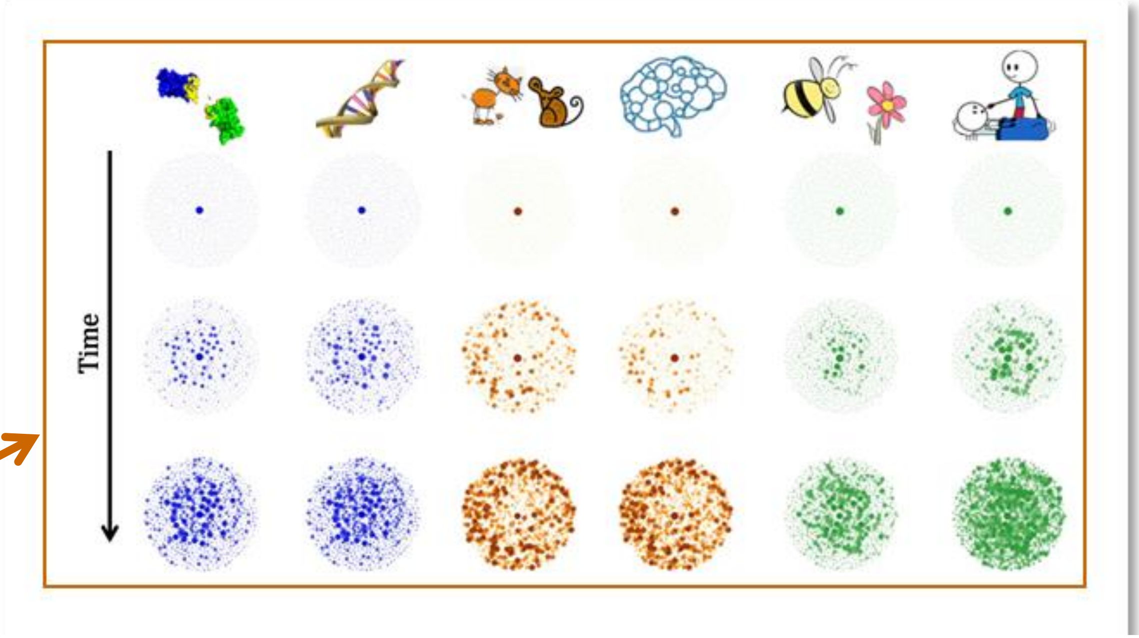
Hubs
=
Free flow





Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

Network GPS – how signals navigate the network

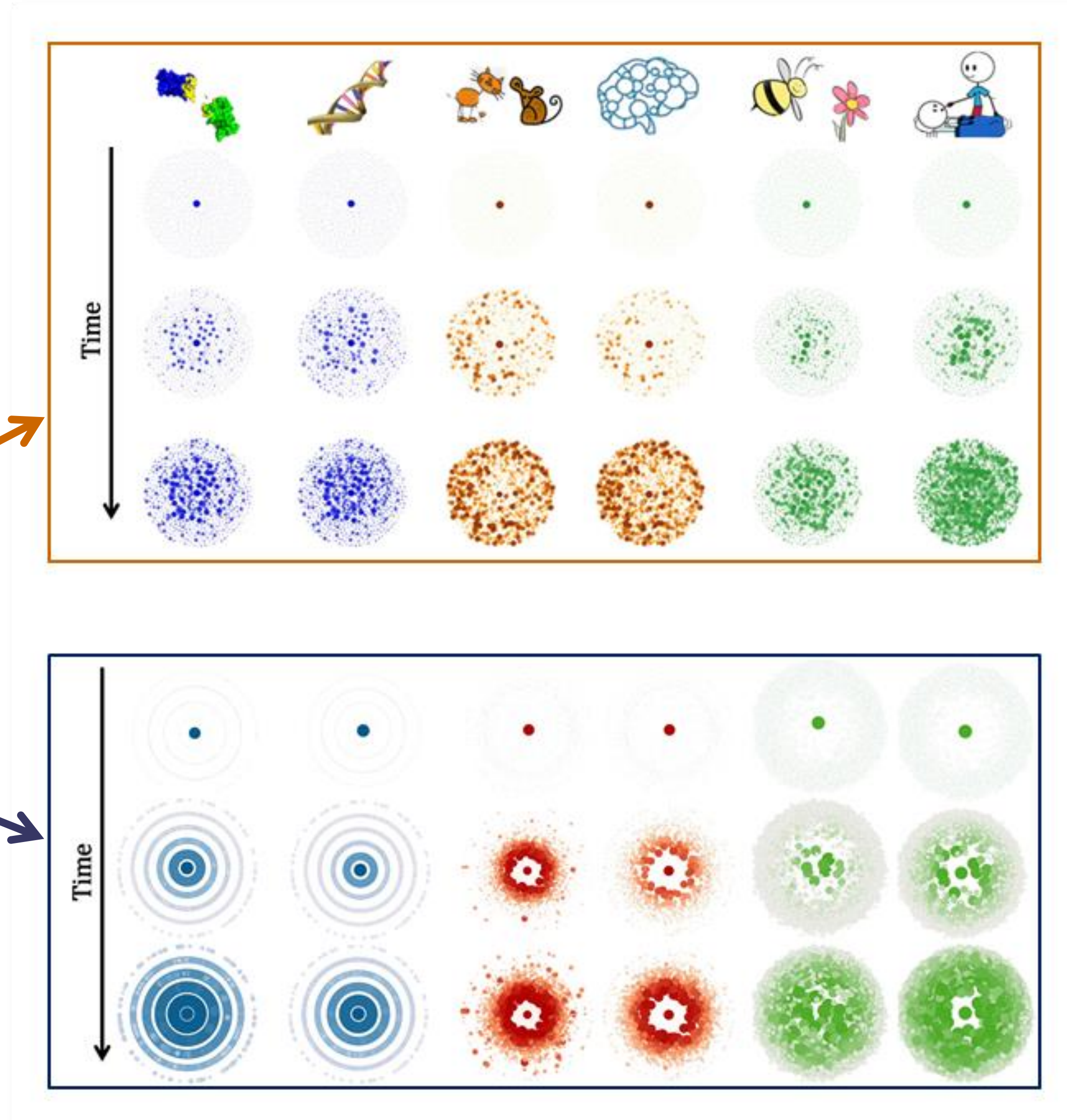


Naïve layout

Network GPS – how signals navigate the network

Naïve layout

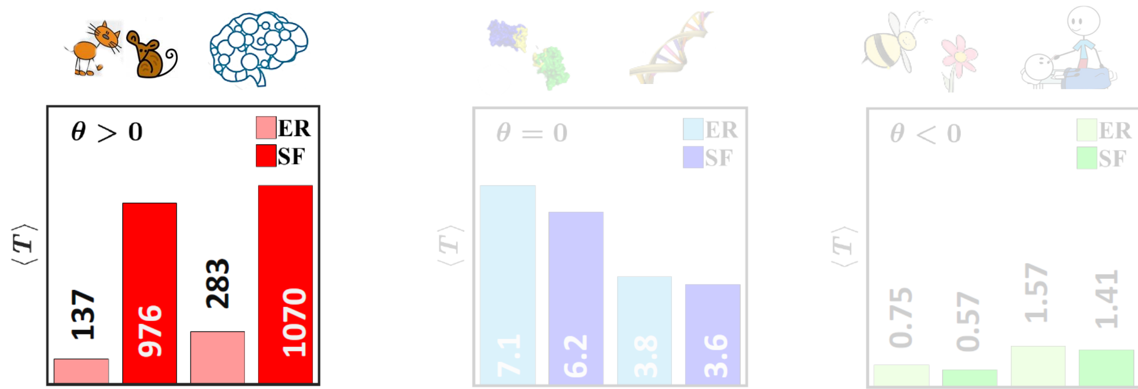
Our GPS prediction



Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

Same topology – different spreading rules

$\boldsymbol{\theta > 0}$
Hubs dramatically slow down propagation



Hubs speed up the propagation

Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

Same topology – different spreading rules

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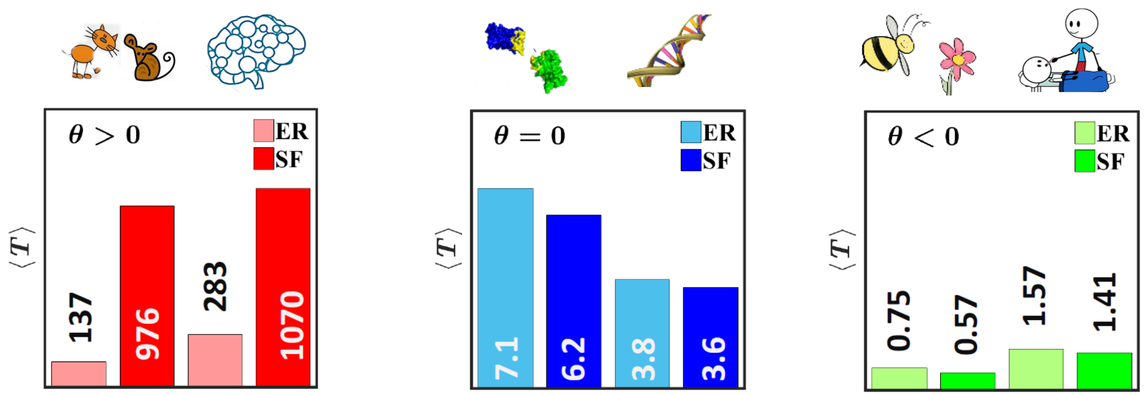
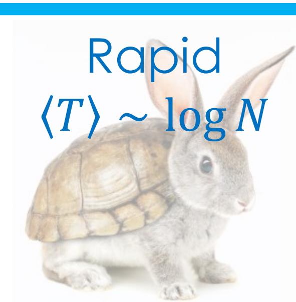


$\boldsymbol{\theta < 0}$
Hubs speed up the propagation

Ultra-slow
$\langle T \rangle \sim N^{\alpha}$



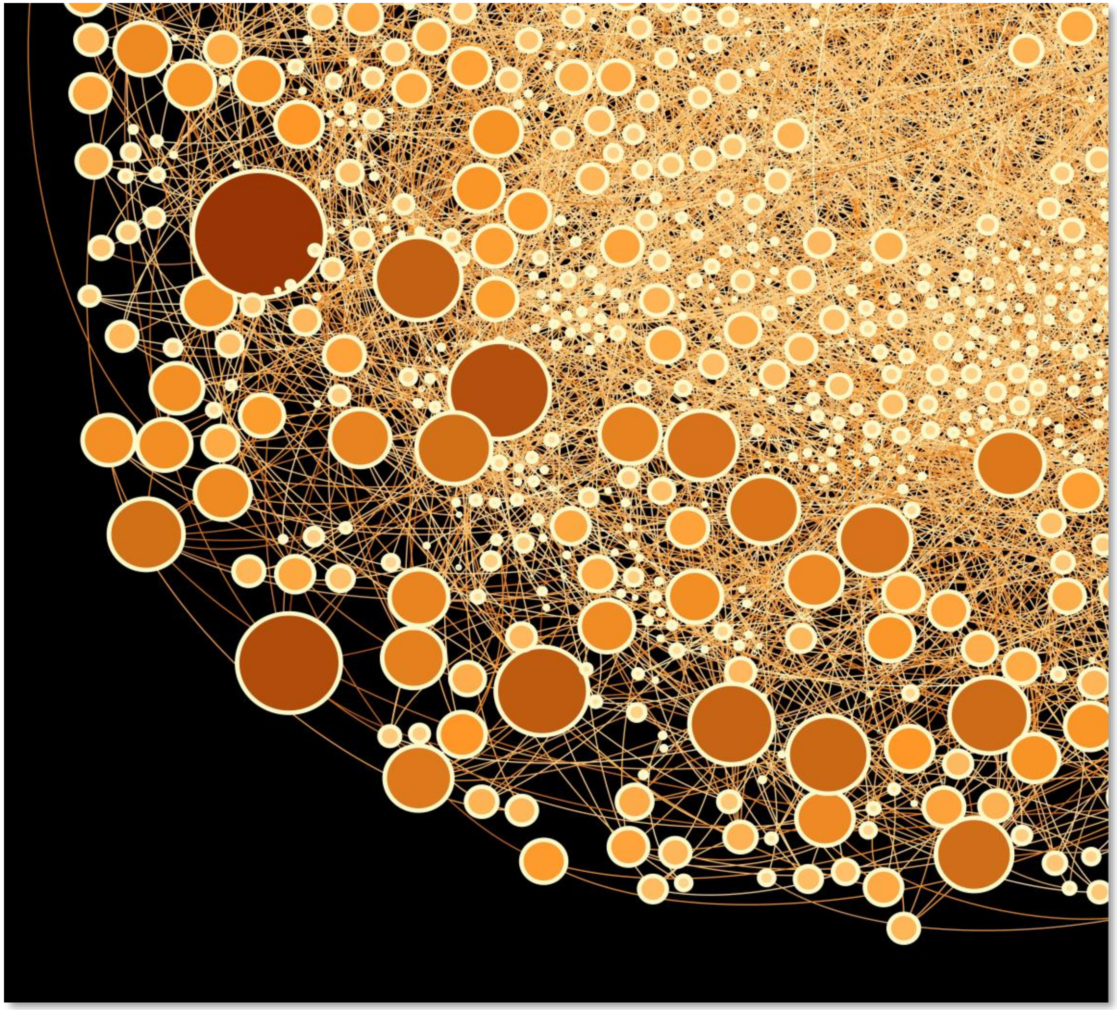
Rapid
$\langle T \rangle \sim \log N$

Ultra-fast
$\langle T \rangle \sim \mathrm{Const}$

Different interpretations of scale-freeness. All boiled down to a single analytically predictable parameter θ

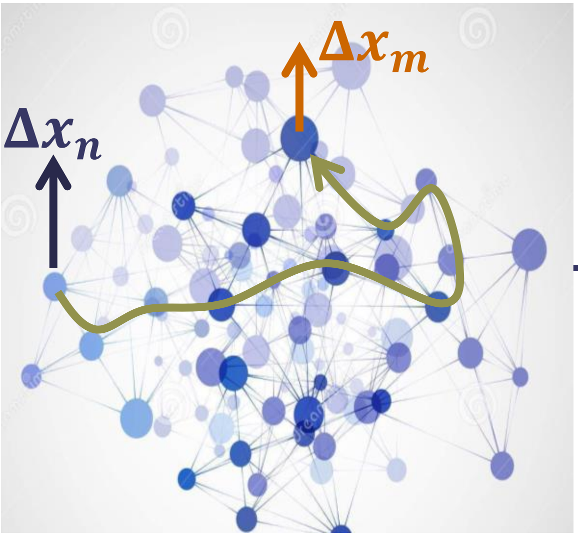
Hens, Harush, Haber, Cohen & Barzel. *Nature Physics* **15**, 403 (2019)

Flow



U. Harush and B. Barzel. *Nature Communications* **2181** (2017)

Information flow



$x_n + \Delta x_n \longrightarrow x_i + \Delta x_i(t) \longrightarrow x_m + \Delta x_m(t)$

signal mediation response

Node flow $\mathcal{F}_i$: how effective is each node in *transferring* information?

Link flow $\mathcal{F}_{ij}$: How effective is each link, pathway?

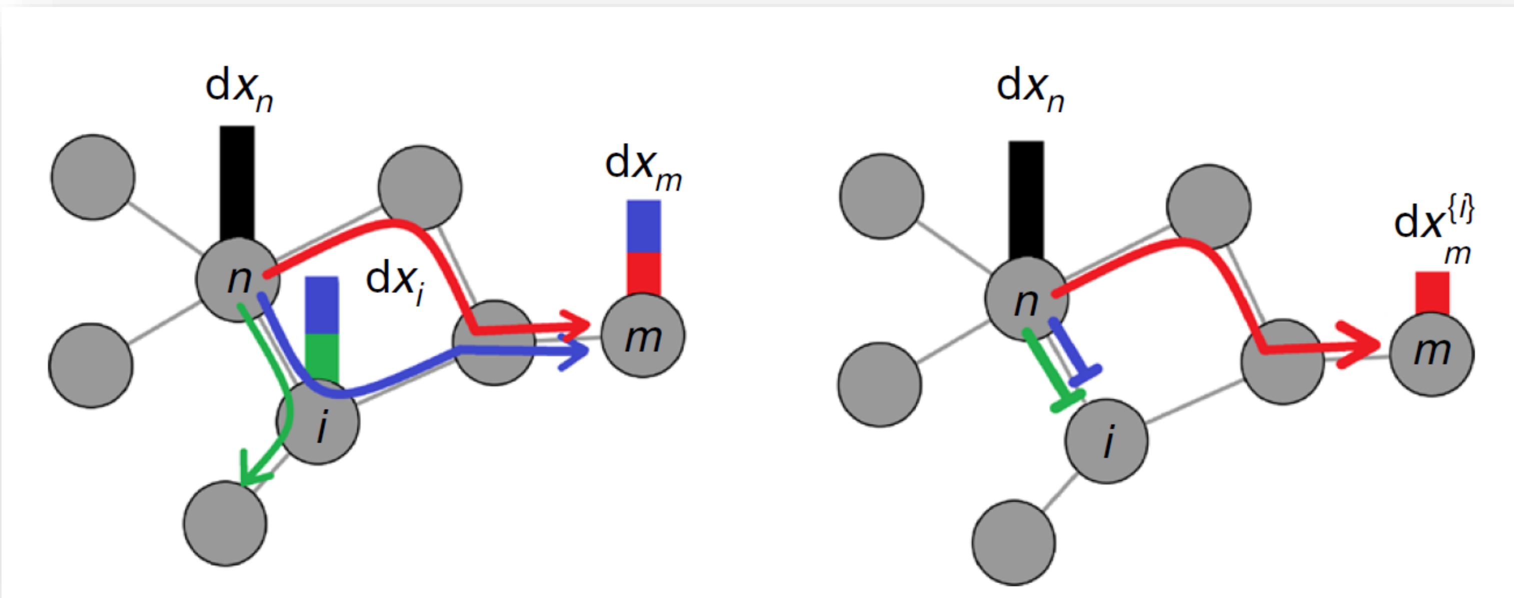
U. Harush and B. Barzel. *Nature Communications* **2181** (2017)

Information flow

Freezing

$\boldsymbol{x_n + \Delta x_n} \longrightarrow \boldsymbol{x_i + \Delta \cancel{x_i}(t)} \longrightarrow \boldsymbol{x_m + \Delta x_m^{\{i\}}(t)}$

signal mediation response



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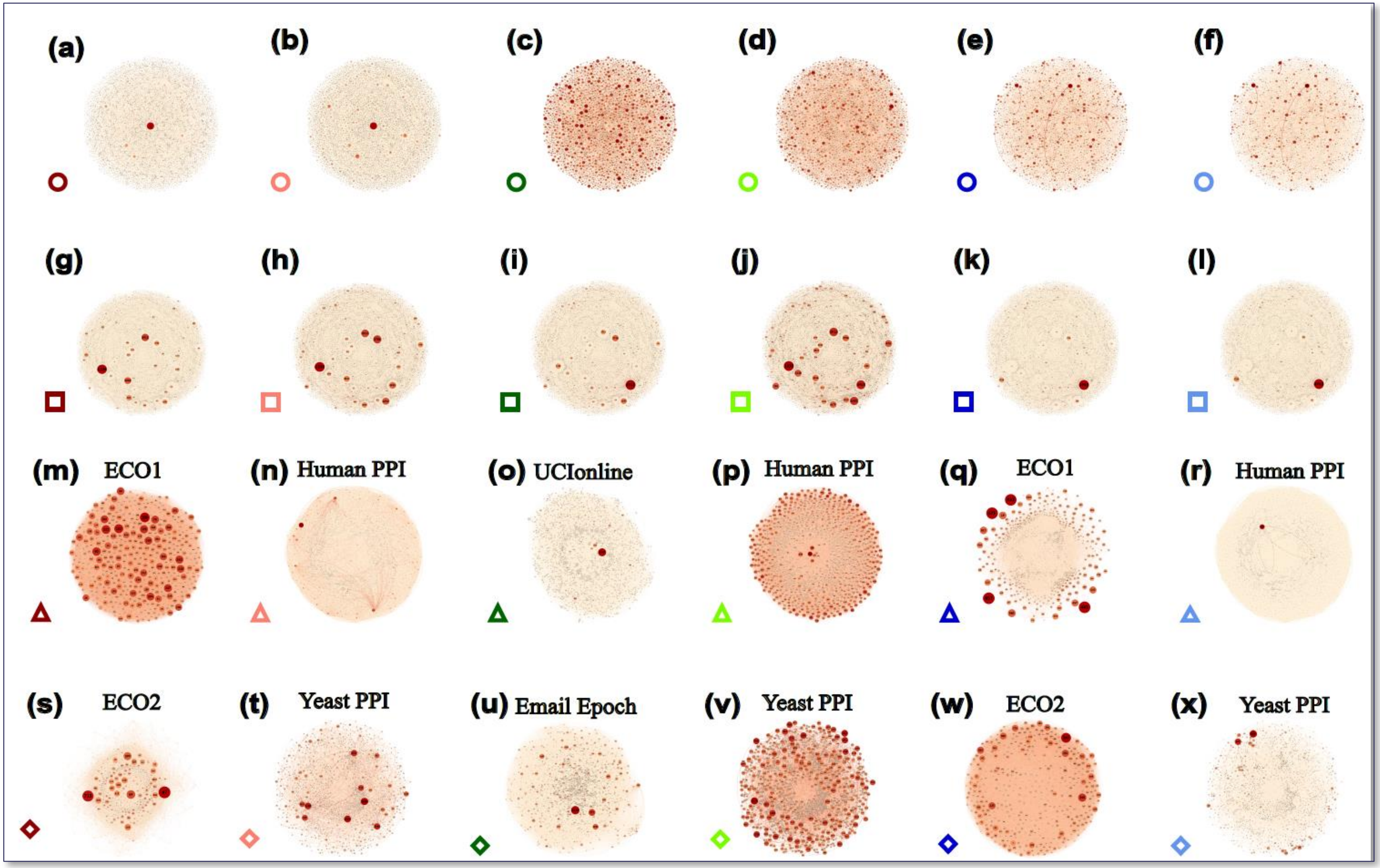
The zoo of information flow patterns



U. Harush and B. Barzel. *Nature Communications* **2181** (2017)

Taming the zoo of information flow patterns

$$\mathcal{F}_i \sim k_{i,\text{out}} k_{i,\text{in}}^{\omega-1}$$

$$\mathcal{F}_{ij} \sim A_{ij} k_{i,out} k_{i,in}^{\xi-1} k_{j,\text{in}}^{\xi}$$

Node/link flow
Dynamic function of interest.

$$\mathcal{F}_i, \mathcal{F}_{ij}, k_{in/out}, \omega, \xi$$

Dynamic determinants
Mapping topology into dynamics

Weighted in/out degrees
Known topological elements

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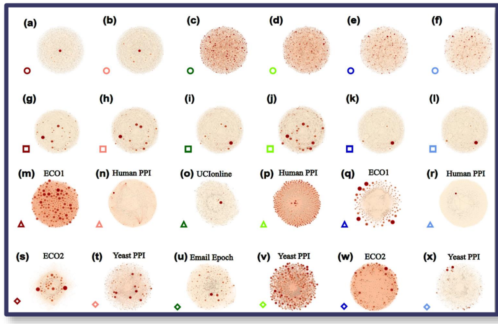
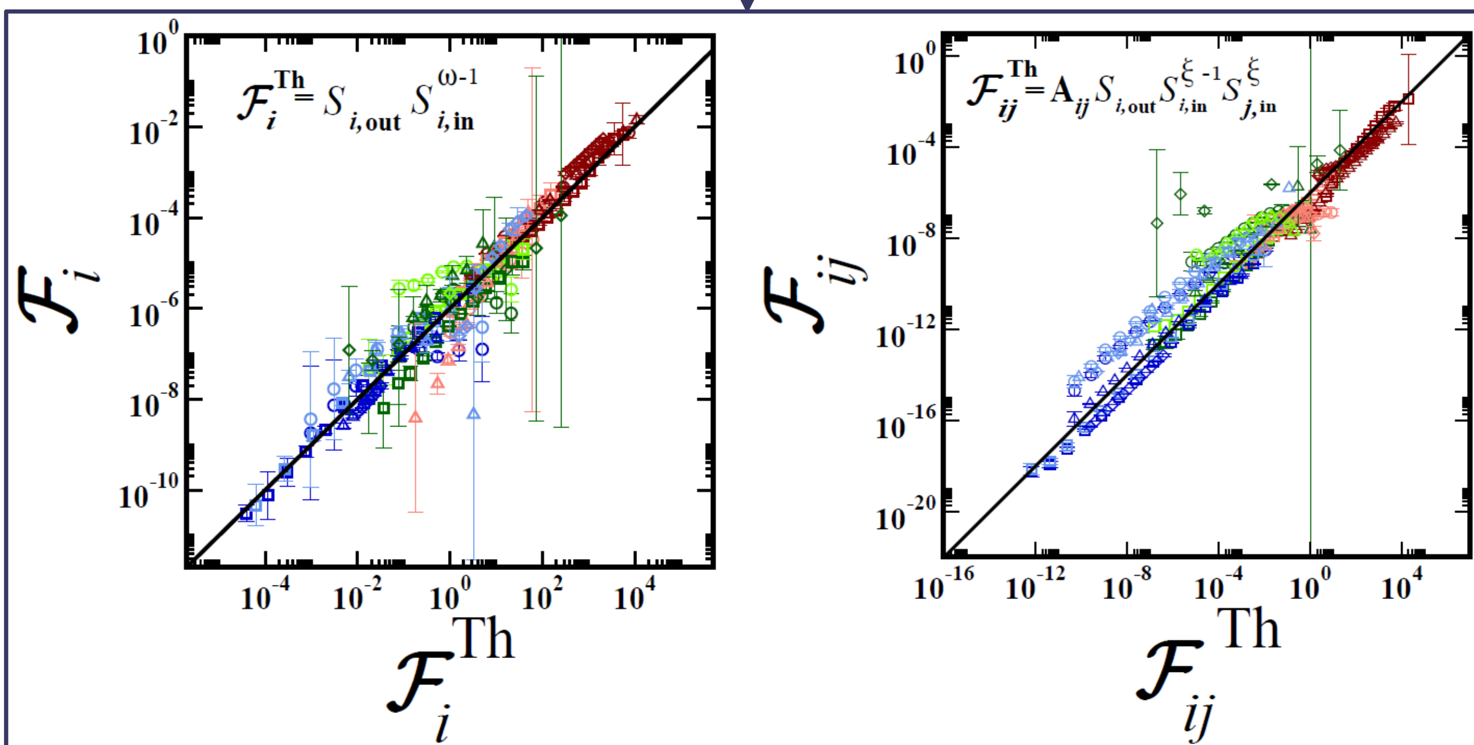
~~The zoo~~ universal information flow patterns



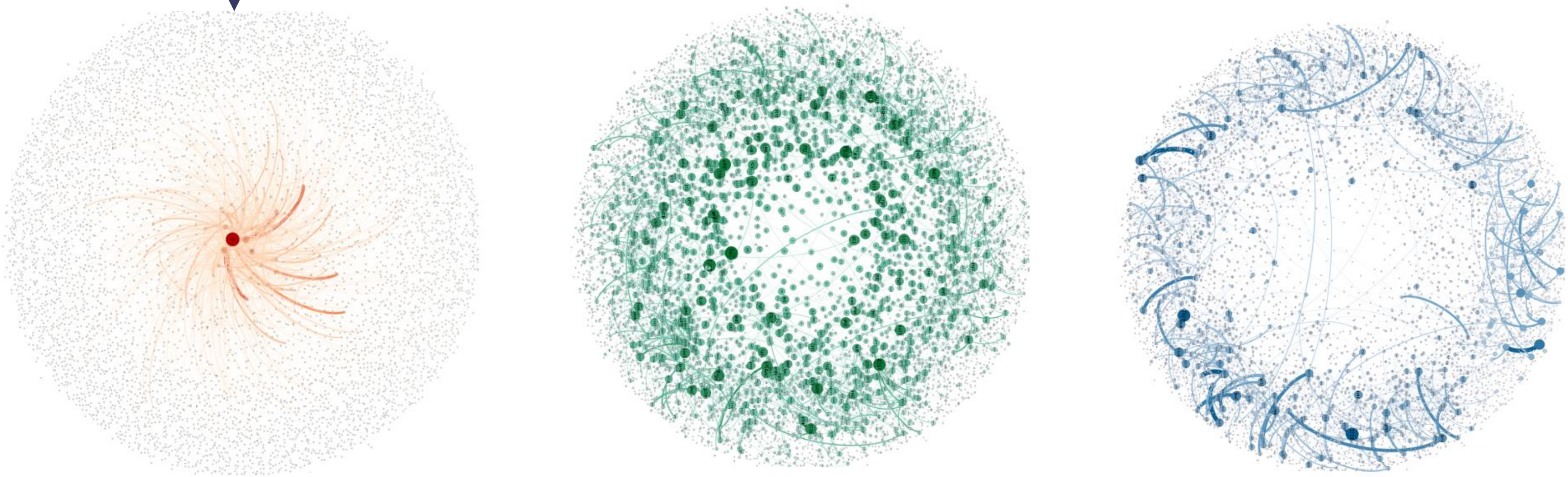


U. Harush and B. Barzel. *Nature Communications* **2181** (2017)

Universal classes of information flow patterns

$$\mathcal{F}_i \sim k_{i,\mathrm{out}} k_{i,\mathrm{in}}^{\omega-1}$$

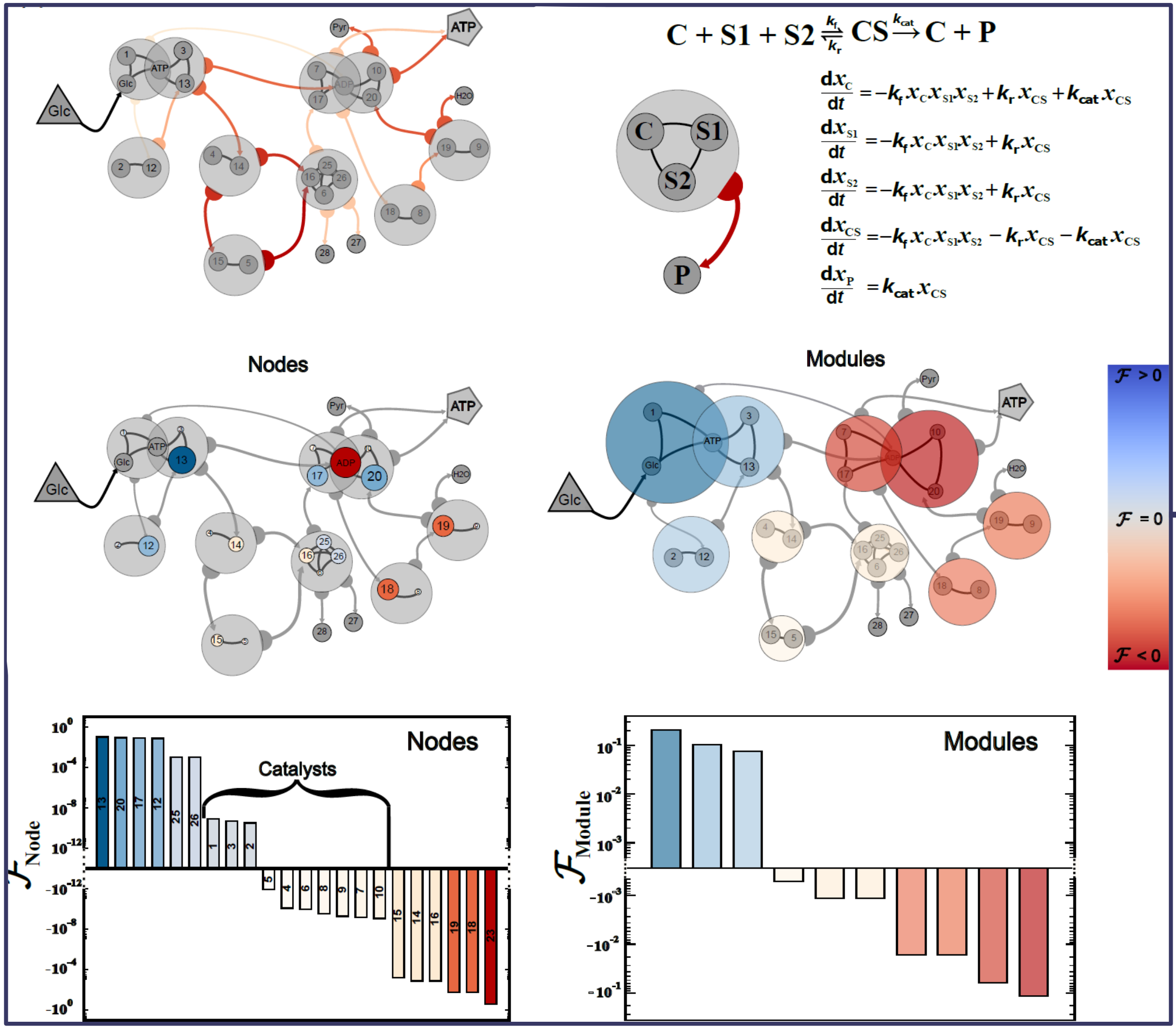
$$\mathcal{F}_{ij} \sim A_{ij} k_{i,out} k_{i,in}^{\xi-1} k_{j,\mathrm{in}}^{\xi}$$



Hub centric | Egalitarian | Peripheral

U. Harush and B. Barzel. *Nature Communications* **2181** (2017)

Flow analysis in Glycolysis: what is a biological network?



Metabolism: Designed to push the mean information flow towards zero.

Output (response) independent of Input (perturbation)

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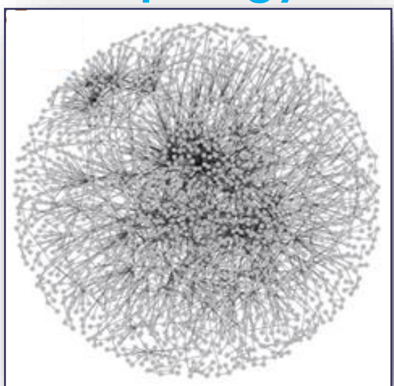
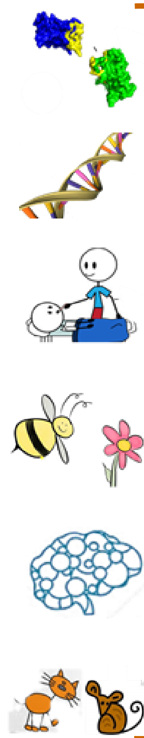
Dictionary of network dynamics

Topology



Interaction mechanisms



Dynamic determinants

$$\tau_i \sim k_i^{\theta}$$

$$I_i \sim k_i^{\varphi}$$

$$Q_i \sim k_i^{\delta}$$

$$\Gamma(l) = e^{-\beta \alpha l}$$

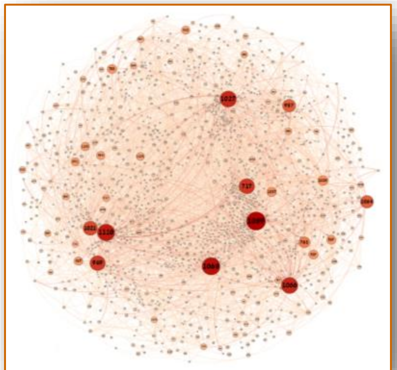
$$C_i \sim k_i^{\omega}$$

$$\beta_{\text{eff}} = \frac{\mathbf{1}^\top A^2 \mathbf{1}}{\mathbf{1}^\top A \mathbf{1}}$$

$$\mathcal{F}_i \sim S_{i,\text{out}} S_{i,\text{in}}^{\xi}$$

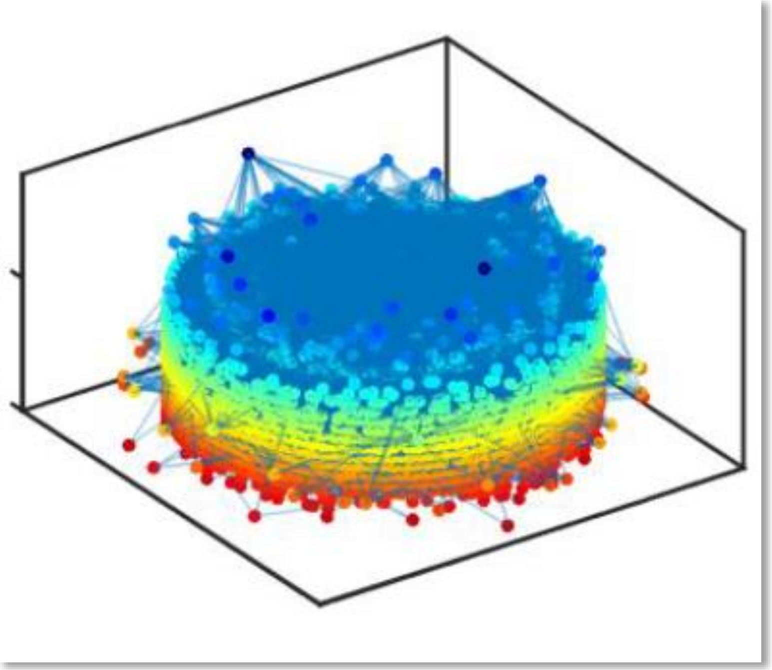
$$\mathcal{F}_{ij} \sim S_{i,out} S_{i,in}^{\xi-1} S_{j,\text{in}}^{\xi-2}$$

Dynamics



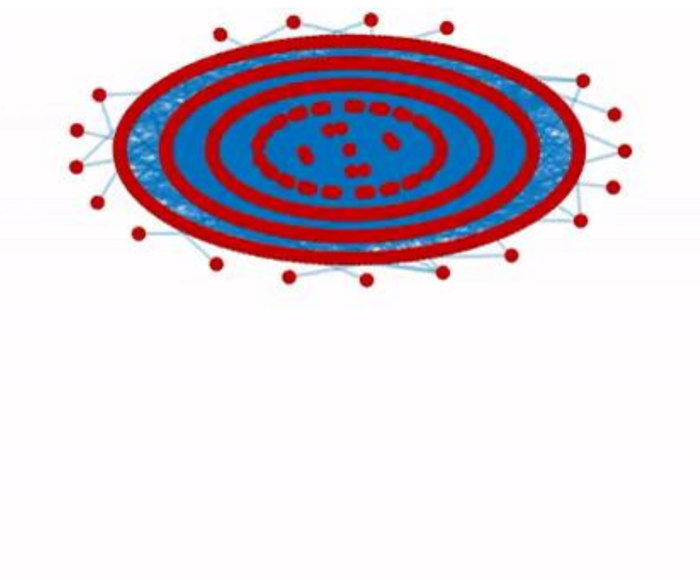
Microscopic Diversity condenses into a discrete set of Universality classes that determine how Topology translates into Dynamics

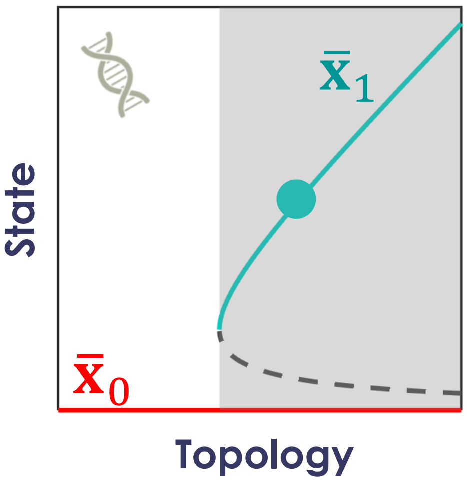
Recoverability

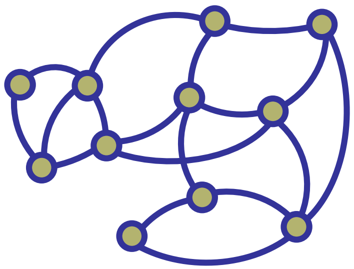


H. Sanhedrai, J. Gao, M. Schwartz, S. Havlin and B. Barzel. *Nature Physics* (In press)

Dynamic transitions







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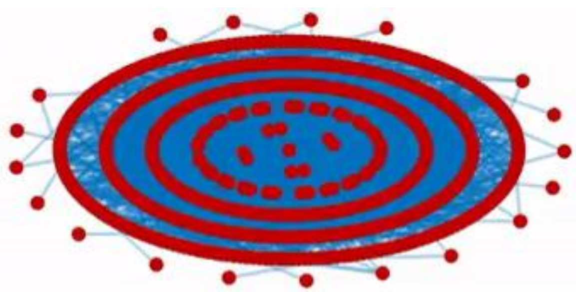
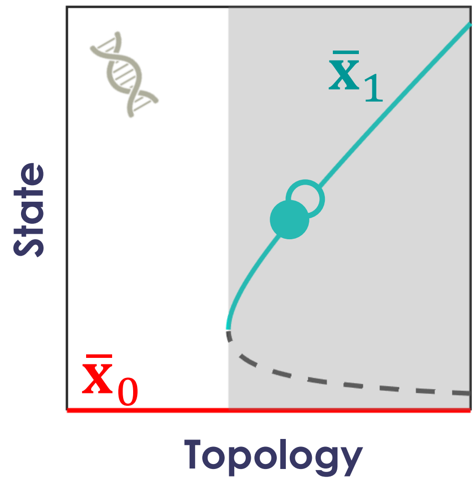
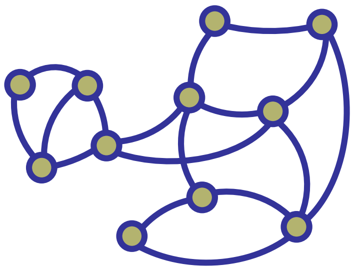
Dynamic transitions







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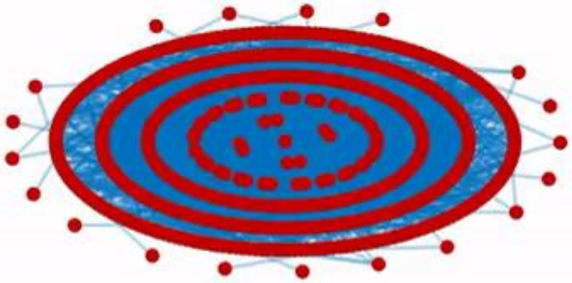

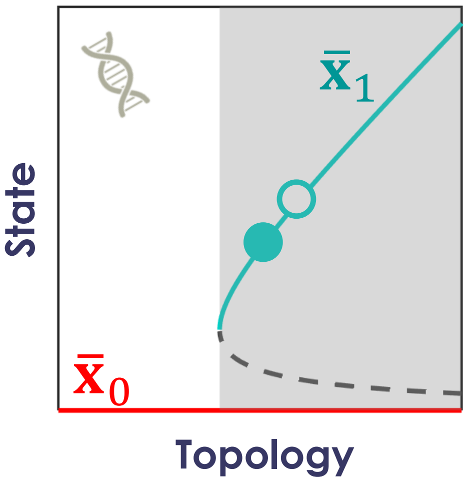

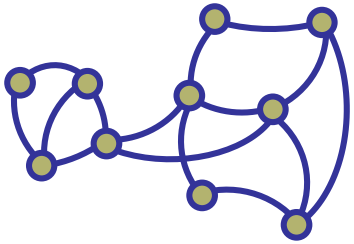

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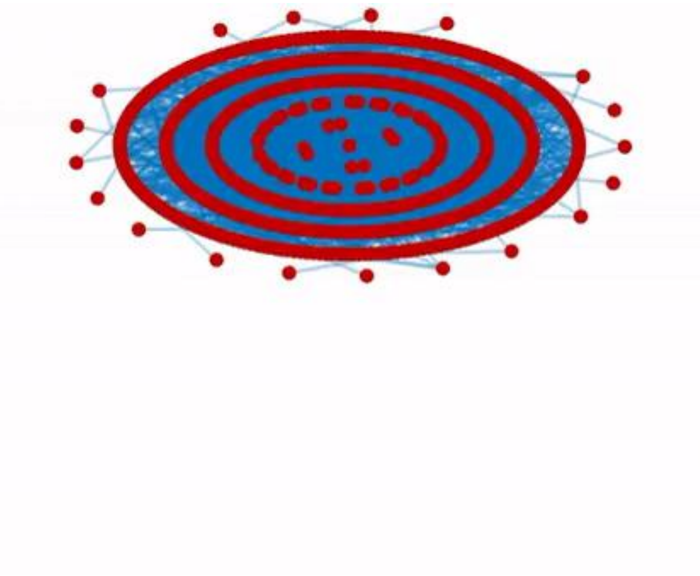

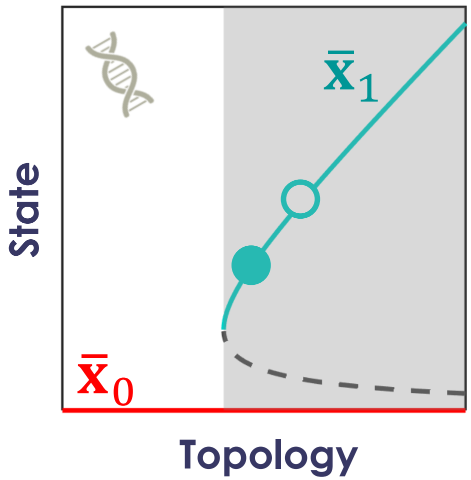

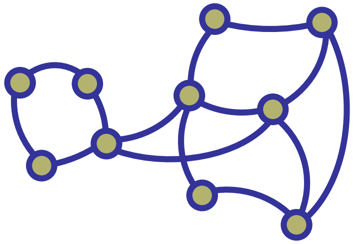

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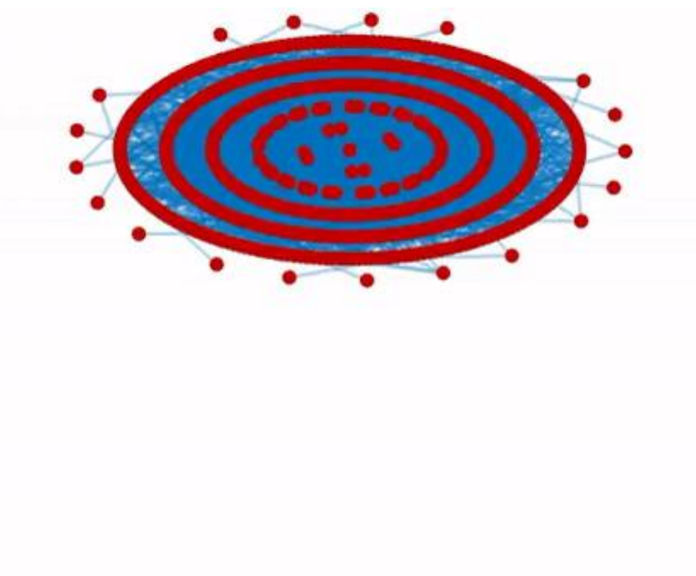

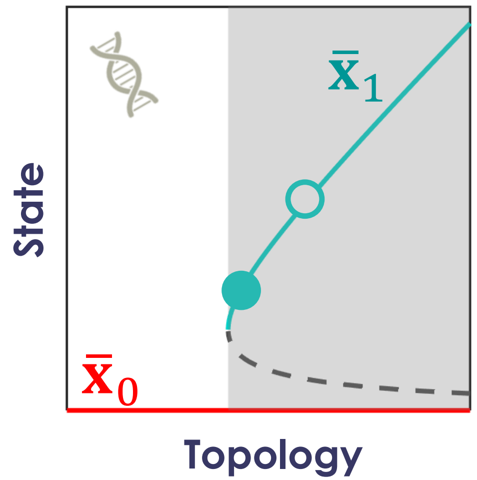

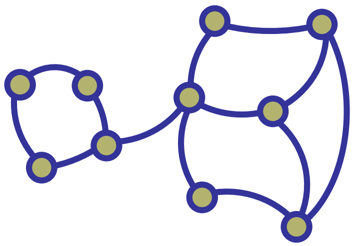

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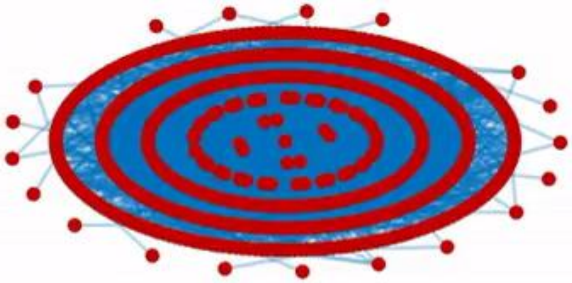

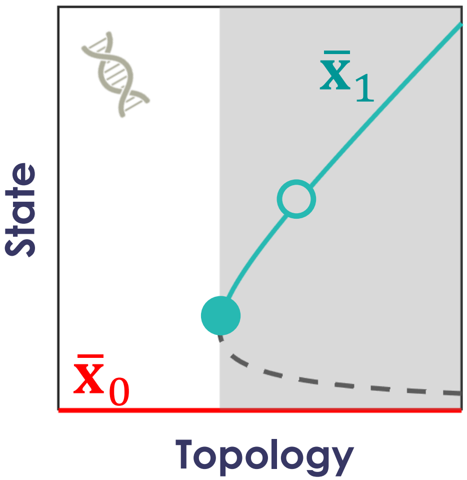

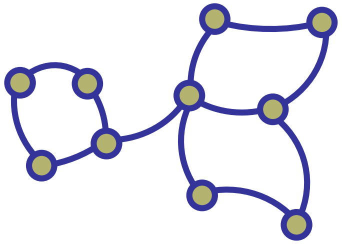

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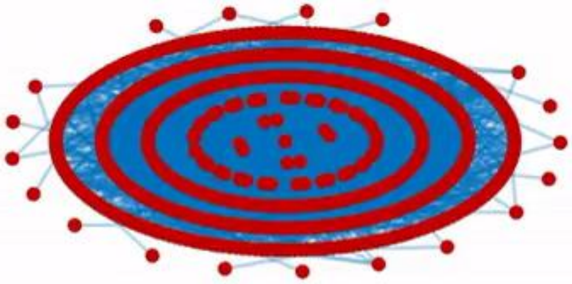

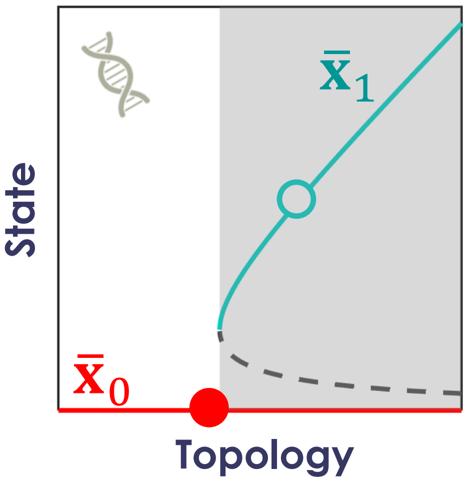

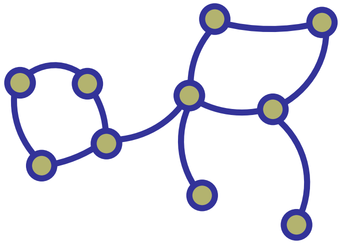

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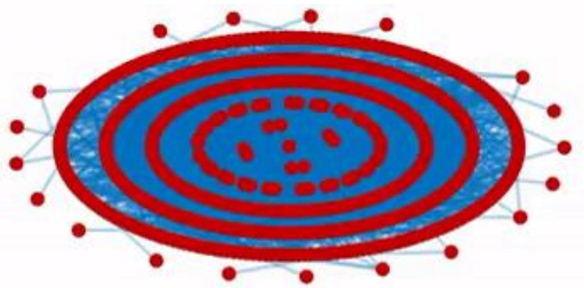
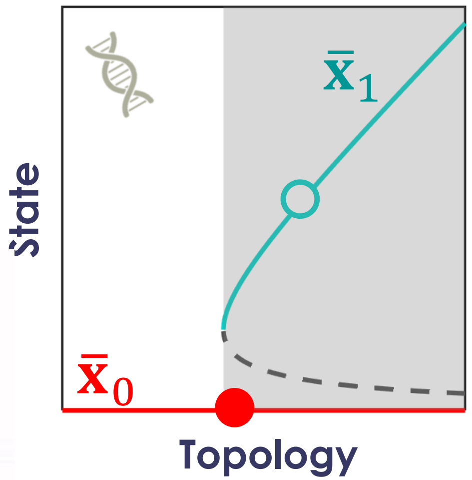
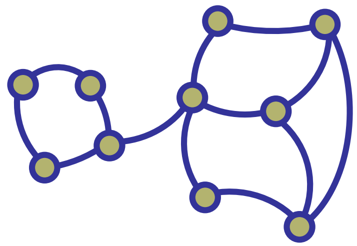
Dynamic transitions - irreversible







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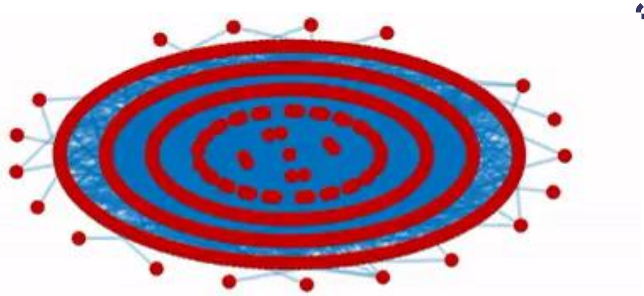
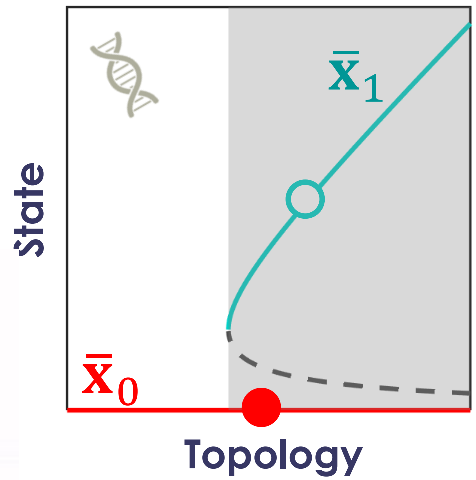
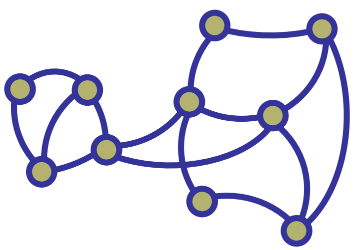
Dynamic transitions - irreversible







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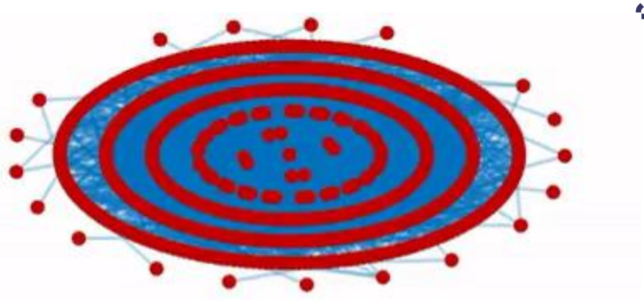

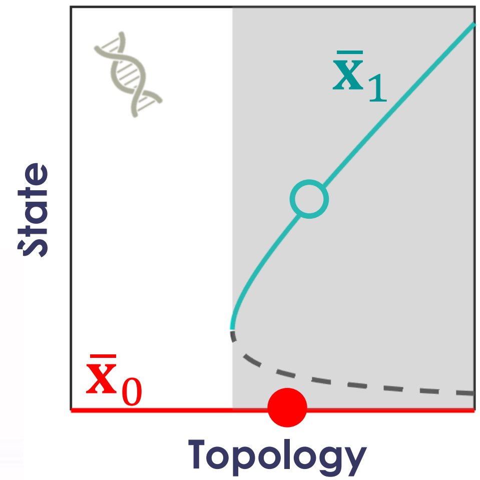

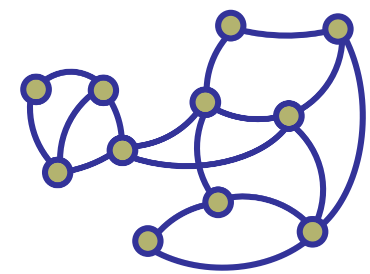

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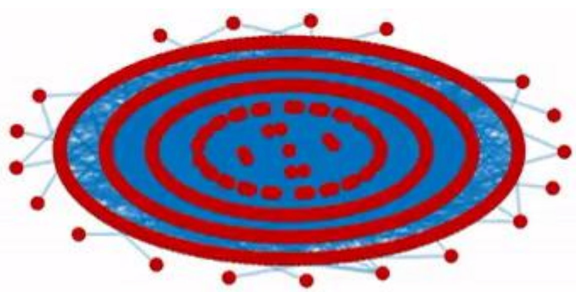
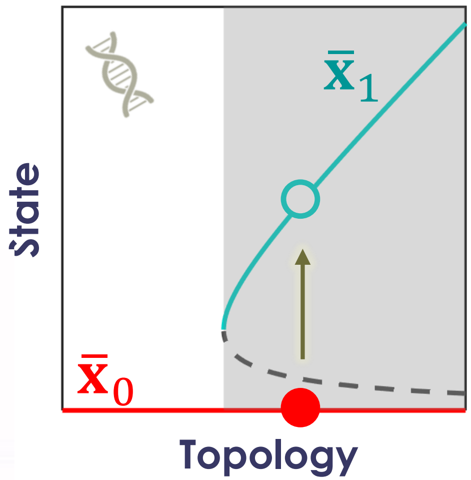
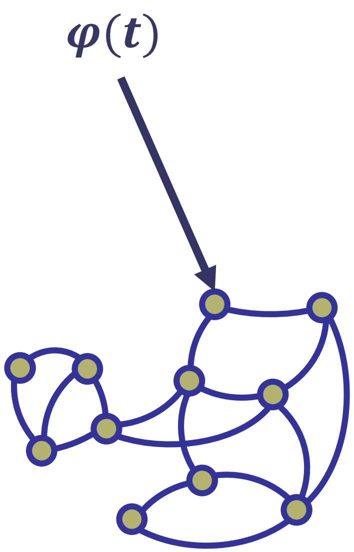
Reigniting the network activity







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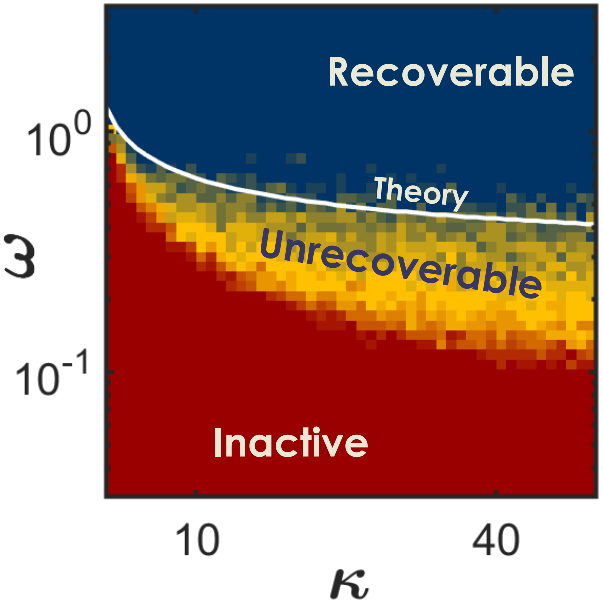
The recoverable phase



Can you reignite a failed system by controlling just one node?

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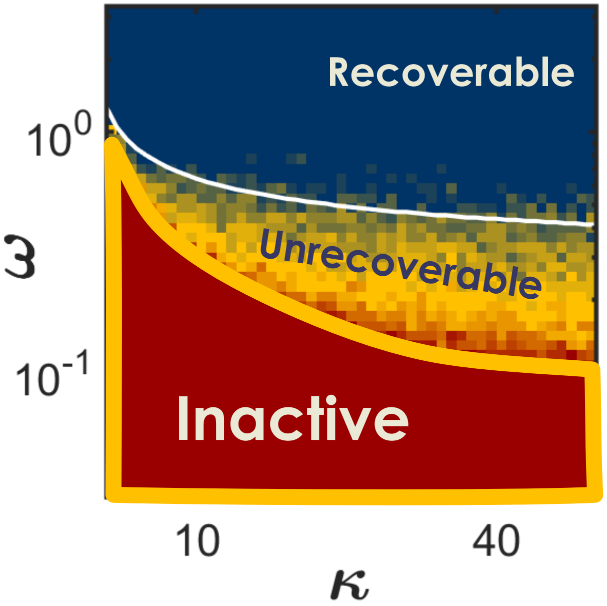
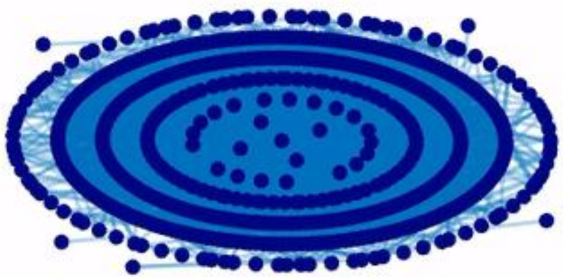
The recoverable phase





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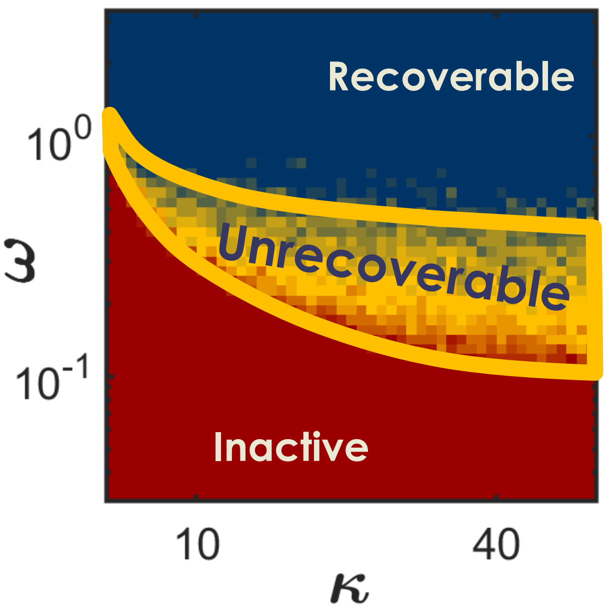

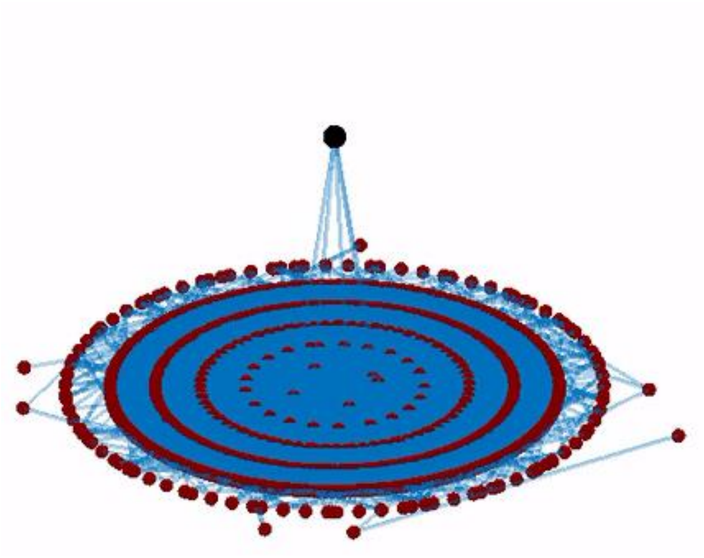

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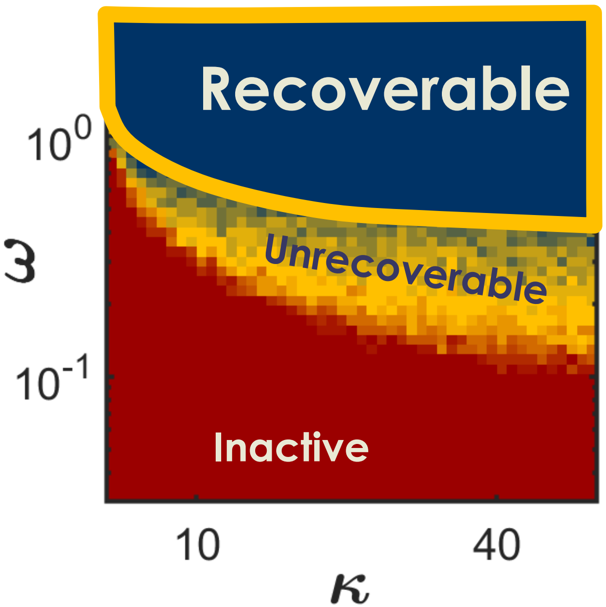
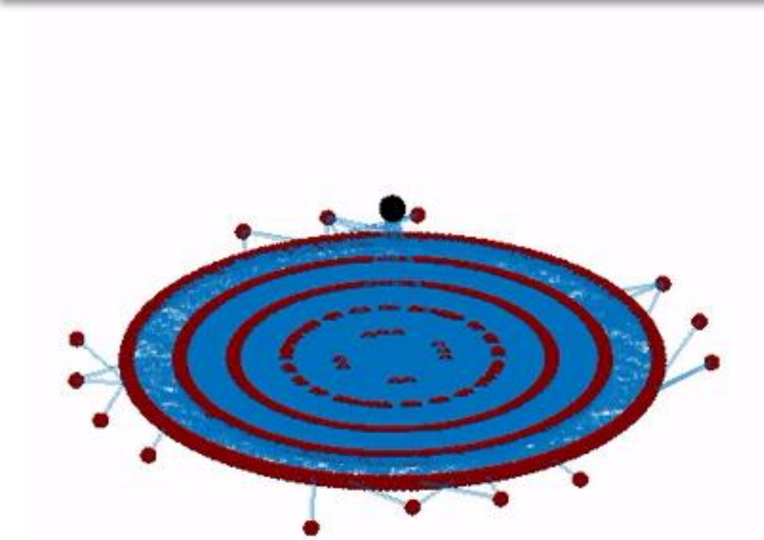


Single node reigniting – reviving the failed system by activating one node



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Theory of network dynamics was brought to you by

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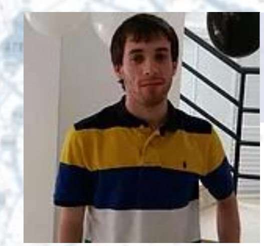

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